Liquid Metal Natural Convection from Plane Surfaces: A Review Including Recent Sodium Measurements

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The design of various components of Liquid Metal Cooled Fast Breeder Reactors (LMFBR) requires knowledge of the natural convection heat transfer conditions. A review has therefore been made of natural convection heat transfer for low Prandtl number fluids: this includes both theoretical and experimental studies. The importance of the surface orientation is emphasized with reference to vertical, inclined, and horizontal surfaces.

The most recent measurements made by the authors are compared with the analytical predictions and measurements made in other low Prandtl number fluids.

NOTATION

a	half-width of horizontal plate
D	plate diameter
g	gravitational acceleration
h_x	local heat transfer coefficient at x
$\bar{h}$	mean heat transfer coefficient
k	fluid thermal conductivity
L	plate length
q	surface heat flux
x	distance from leading edge
z	appropriate dimension for geometry, i.e., a, or D, or L, or x
Gr_z	Grashof number based on z $(g\beta_{th}\theta z^3/\nu^2)$
Gr_z^*	modified Grashof number based on z $(g\beta_{th}qz^4/k\nu^2)$
Nu_x	local Nusselt number $(h_x x/k)$
$\overline{Nu}$	mean Nusselt number $(\bar{h}z/k)$
Pr	Prandtl number (ν/α)
α	fluid thermal diffusivity
β_{th}	volumetric expansion coefficient
ν	fluid kinematic viscosity
θ	temperature difference, wall-to-bulk fluid

1 INTRODUCTION

In some regions of the large sodium pools of LMFBRs the flow velocities are such that the dominant heat removal mechanism is that of natural convection. Consequently there are components which have their operating temperatures determined by natural convective heat transfer coefficients. Typically in this context is the inner tank of a pool-type LMFBR (Fig. 1), where the main function of the component is to separate the high temperature core outlet sodium flow from the cooler sodium discharged from the intermediate heat exchangers. This primarily involves heat exchange by natural convection from a vertical surface, but for some components different orientations are encountered. For example, the design of internal core debris trays (Fig. 1) involves the use of near horizontal surfaces with the heat rejection downwards to the main sodium. This review attempts to deal with the differences produced by the various orientations of the heat transfer surface.

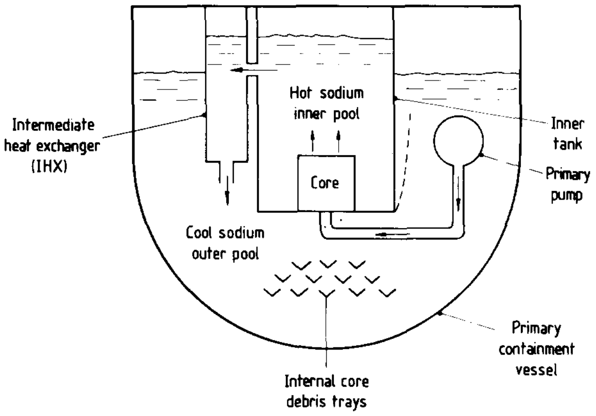

Fig. 1. Simplified schematic arrangement of a pool-type LMFBR

Analysis plays a most important role in the review partly because of the limited experimental data for liquid metals, but also because of its particular relevance to low Prandtl number fluids. The latter point is analogous to the forced flow situation where significant progress can be made by simple analysis even when the flow is turbulent: this is possible because of the dominant influence of the molecular conductivity, which persists up to moderate Reynolds numbers for particularly low Prandtl number fluids like sodium. As will be seen there is practically no turbulent natural convection data available for liquid metals but it is possible that a limited extrapolation of the laminar analyses will be acceptable, particularly for sodium. Recent sodium measurements by the authors (**1**) are included for comparison with the theoretical predictions.

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2 VERTICAL SURFACES

2.1 Laminar Flow Analysis for Low Prandtl Number Fluids

In general three methods have been used to analyse the natural convection from vertical surfaces in laminar flow, these are:

(1) Integral method
(2) Similarity method
(3) Perturbation method

The first two methods make simplifications to the basic continuity, momentum, and energy equations involving classical boundary layer assumptions. Of these the similarity method is more relevant to low Prandtl numbers since in the integral method the velocity and thermal boundary layer shapes have to be assumed. The similarity method uses a transformation technique to solve the differential equations assuming that the temperature (and velocity) profiles have similar shapes at different positions along the plate. Pohlhausen et al. (**2**) were the first to use the method but it was Ostrach (**3**) who extended its use to a large range of the Prandtl number (0·01–1000).

The similarity method has been widely applied since, and both uniform heat flux and isothermal wall boundary conditions have been studied. For low Prandtl numbers Sparrow and Gregg (**4**) obtained the correlations given in Table 1 for isothermal wall conditions.

Table 1

Predicted natural convection correlations at low Prandtl numbers (**4**)

Pr	$\frac{Nu_x}{(Gr_x Pr^2)^{1/4}}$
0·03	0·5497
0·02	0·5582
0·008	0·5729
0·003	0·5827
0	0·6004

(The limiting solution for $Pr \to 0$ was found by Le Fevre (**5**))

It is interesting to note that the correlation is based on $(Gr_x Pr^2)$, which is obtained by dimensional analysis if the viscous terms are neglected. The large boundary layer thickness for low Prandtl numbers produces such conditions.

Applying the above to sodium at reactor conditions ($Pr \sim 5 \times 10^{-3}$)

$$Nu_x = 0{\cdot}5765(Gr_x Pr^2)^{1/4} \tag{1}$$

Note that the mean Nusselt number for a plate of length x is 1·33 Nu_x in this case.

For uniform heat flux conditions Sparrow and Gregg (**6**) only went down to $Pr = 0{\cdot}1$, but more recent evaluations by Chang (**7**) *et al.*, present results which when extrapolated to sodium give

$$Nu_x = 0{\cdot}7320(Gr_x^* Pr^2)^{1/5} \tag{2}$$

Note that the mean Nusselt number for a plate of length x is 1·2 Nu_x in this case.

As noted above, the use of the similarity method makes certain classical boundary layer assumptions based upon the boundary layer thickness being small relative to the distance from the leading edge. Because of the thick boundary layers associated with low Prandtl number fluids these assumptions are brought into question. Sparrow and Guinle (**8**) have thus used the perturbation method, which makes no simplification of the basic equations, to consider the effects of the additional terms on the local surface heat flux for an isothermal plate. These authors calculated the Grashof number delineating the threshold of significant (5 per cent) deviations from the classical boundary layer results. At $Pr = 0{\cdot}003$ this threshold was found to be $Gr_x < 1{\cdot}5 \times 10^6$: a value achieved for $x \sim 10$ mm in sodium with a surface-to-bulk temperature difference of 50° C. Thus the assumptions made in the similarity method relating to the boundary layer are confirmed for practical situations.

2.2 Measurements in Low Prandtl Number Fluids

Inevitably the low Prandtl number fluids are liquid metals and the technical problems associated with such fluids have tended to restrict the number of experiments compared with more conventional fluids. Further, the majority of the experimental studies have used mercury ($Pr \sim 0{\cdot}025$) rather than sodium ($Pr \sim 0{\cdot}005$).

Several experiments with mercury have been reported, but the most recent come from two sources. In the first, by Julian and Akins (**9**) and Chang and Akins (**10**), a uniform heat flux boundary condition was established on a 50 mm long plate. A range of modified Grashof number up to 10^9 was possible, and the results indicate values of Nu_x greater than the similarity predictions over the majority of this range. There is some indication at the lower Gr_x^* of the effects predicted by the perturbation method because of axial heat conduction in the fluid whereas at the higher Gr_x^* the deviation from the similarity predictions is small.

A substantial experimental programme has been pursued by Welty's team at Oregon State University, USA, during the past few years, and their work (**11–14**) forms the second source of recent mercury data. Uniform heat flux conditions have been used with plates 38 mm and 147 mm long, and also vertical cylinders were studied for the same boundary conditions. The range of operating conditions in their various experiments are summarized in Table 2.

Possibly the most reliable data was obtained on the 147 mm long vertical plate, since this study could benefit from the earlier experiments. The correlation obtained was identical to that of Julian and Akins (**9**) and as such gives Nu_x slightly (± 7 per cent) different than predicted by the similarity theory: the theory is lower at the lower Gr_x^* and vice versa at the higher values of Gr_x^*. All of the experimental correlations are compared with the predictions given by the similarity method in Fig. 2.

The influence of the Prandtl number on the transition Grashof number is not well established for low Prandtl number fluids. Wiles and Welty (**13**) stated that previous work had reported transition values of $Gr_x^* \sim 5 \times 10^9$.

Table 2

Range of operating conditions in Oregon mercury experiments

Experimental configuration	Range of modified Grashof No. Gr_x^*	Comments
Vertical plate 38 mm long (**11**)	10^5–10^8	$Nu_x <$ Chang and Akins by 5–15 per cent
Vertical plate 147 mm long (**14**)	10^6–10^{11}	Nu_x different than similarity theory by ±7 per cent

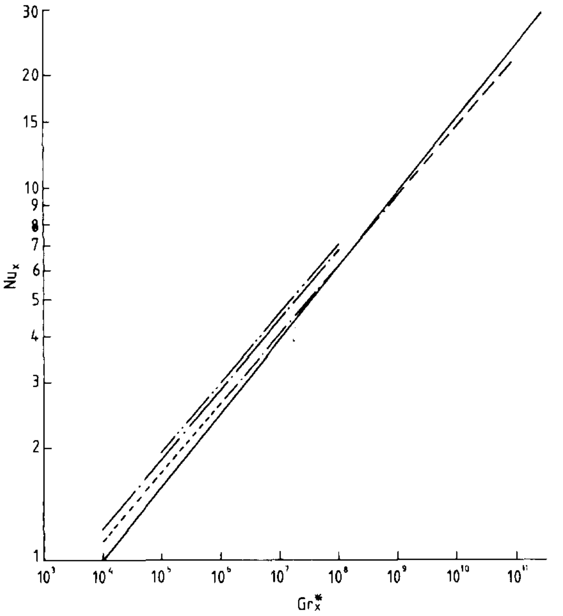

Fig. 2. Comparison of mercury experiments with similarity theory for a vertical plate

			Gr_x^* Range
———	Similarity Theory	$Nu_x \quad 0{\cdot}156\ Gr_x^{*0\cdot2}$	
— — —	Julian and Akins (**9**)	$Nu_x = 0{\cdot}196\ Gr_x^{*0\cdot188}$	10^4–10^9
—·—	Chang and Akins (**10**)	$Nu_x = 0{\cdot}213\ Gr_x^{*0\cdot188}$	10^4–10^8
—··—	White et al (**11**)	$Nu_x = 0{\cdot}226\ Gr_x^{*0\cdot187}$	10^5–10^8
—— ——	Humphreys and Welty (**14**)	$Nu_x = 0{\cdot}196\ Gr_x^{*0\cdot188}$	10^6–10^{11}

This corresponds to a transition $Gr_x \sim 4 \times 10^8$. Assuming that transition occurs at the same local Gr_x on a uniform heat flux plate as on an isothermal plate, this value of 4×10^8 is the same order of magnitude as the transition Grashof numbers observed for fluids with $Pr \sim 1$. This suggests that the criterion for transition from laminar flow may be a critical Grashof number, and may be almost independent of the Prandtl number.

3 DOWNWARD-FACING SURFACES

3.1 Laminar Flow Analysis for Horizontal Surfaces

The case of finite horizontal downward-facing surfaces is a peculiar phenomenon in natural convection since the hydrostatic pressure gradient in the direction normal to the surface is first induced, and then its horizontal gradient acts as the direct driving force for convection. In most natural convection situations the buoyancy force component tangential to the surface of a body acts as the convective driving force.

It has been shown that a similarity solution cannot be found for the boundary layer below a horizontal heated surface, and therefore all the analyses have used forms of the approximate integral method. The differences in the treatments lie in the definition of the boundary condition at the plate edges. Experiments have shown that the boundary layer decreases from a maximum thickness at the plate centreline to a minimum, but finite, thickness at the plate edges. Some early analyses, however, for ease of mathematical solution, assumed a zero boundary layer thickness at the plate edge. Recent papers (**15**, **16**) have tried to take into account the finite thickness at the plate edge. These are each based upon a different physical concept, but they all result in a critical thickness at the plate edges.

Clifton and Chapman (**15**) obtained solutions for the mean Nusselt number for a range of Prandtl numbers with isothermal wall conditions. For sodium at reactor conditions ($Pr \sim 5 \times 10^{-3}$), the following is obtained for isothermal horizontal downward-facing surfaces

$$\overline{Nu} = 0{\cdot}5212(Gr_a Pr^2)^{1/5} \tag{3}$$

For uniform heat flux conditions, Fujii (**16**) *et al.*, obtained corresponding solutions, which, when applied to sodium, give

$$\overline{Nu} = 0{\cdot}5220(Gr_a^* Pr^2)^{1/6} \tag{4}$$

3.2 Influence of Inclinations to Horizontal

The forms of the boundary layer on a vertical and a horizontal downward-facing surface are completely different. On a vertical surface, the boundary layer commences from zero thickness at the leading edge and increases continuously in thickness towards the trailing edge. However, on a horizontal surface facing downwards, the boundary layer has a maximum thickness with a stagnation point at the plate centreline, and the thickness decreases to a finite minimum value at the plate edges. Thus, the form of the boundary layer must change at a critical angle of inclination. However, this critical angle has not been determined theoretically or experimentally.

When a surface is inclined downwards from the vertical, it is predicted theoretically that solutions can be deduced from the formulae for a vertical plate by taking the component of the gravitational force parallel to the plate surface, i.e., ($g \cos \beta$) is substituted for g in the Grashof number, where β is the angle of inclination of the surface to the vertical. For example, eq. (2) becomes

$$Nu_x = 0{\cdot}7320(Gr_x^* \cos \beta Pr^2)^{1/5} \tag{5}$$

A crude estimate of the limiting angle for application of this modification can be made by using the mean heat transfer coefficient for an inclined surface, based on modified vertical plate theory, and evaluating the inclination required to agree with the value predicted for a horizontal surface. For sodium at reactor conditions the critical angle is much less than one degree to the horizontal.

4 UPWARD-FACING HORIZONTAL SURFACES

The analysis of upward-facing horizontal surfaces has received less attention than other configurations, probably because, being a naturally unstable convection situation, transition to turbulent flow takes place at relatively low Grashof numbers and turbulent flow is not amenable to analysis. Because of this, accurate agreement with experimental measurements would not necessarily be expected, and this is seen in the comparisons made below.

For the laminar regime, Pera and Gebhart (**17**) have used the similarity method. Extrapolating their results to sodium at reactor conditions, the following is obtained for isothermal conditions

$$Nu_x = 0{\cdot}480(Gr_x Pr^2)^{1/5} \qquad (6)$$

For the turbulent regime, Levy (**18**) quotes an integral analysis, but the results are not applicable for low Prandtl numbers because they predict a dependence of Nu_x on $(Gr_x Pr^{3/4})$ rather than $(Gr_x Pr^2)$ which is expected. However, Fujii and Imura (**19**) observed in their experiments on upward-facing horizontal surfaces that the heat transfer coefficient agreed with that in the turbulent regime on a vertical plate. On this basis, the improved integral method of Kato (**20**) *et al.*, for turbulent natural convection on a vertical plate can be used. For $Pr = 0{\cdot}005$, this gives

$$Nu_x = 0{\cdot}176\ Gr_x^{0\cdot 32} Pr^{0\cdot 55} \qquad (7)$$

The only reported experimental investigations that have been carried out on natural convection heat transfer to sodium are two experiments with circular horizontal upward-facing surfaces. McDonald and Connolly (**21**) carried out experiments on a cooled horizontal circular plate, 200 mm in diameter, facing downwards in a tank of sodium, which is thermally equivalent to an upward-facing heated plate. For a range of Grashof numbers from 6×10^8 to 5×10^9, the results are correlated by

$$\overline{Nu} = 0{\cdot}262(Gr_D Pr^2)^{0\cdot 35}$$

The index $\sim 1/3$ is characteristic of a turbulent flow regime. This relation gives values of $\overline{Nu}$ approximately 30 per cent higher than those based on eq. (7).

Kudryatsev (**22**) *et al.*, also performed sodium experiments using a 38 mm diameter heated plate. They observed transition from laminar to turbulent flow at $Gr_D = 10^8$. For the laminar regime below $Gr_D = 10^8$, they found that their data were in satisfactory agreement with the empirical relation

$$\overline{Nu} = 0{\cdot}67\left[\frac{(Gr_D Pr^2)}{1 + Pr}\right]^{1/4}$$

This shows positive deviations from the theoretical predictions based on eq. (6) of 0–25 per cent in the range of Gr_D from 10^6 to 10^8. In the turbulent regime above $Gr_D = 10^8$, their data was correlated by

$$\overline{Nu} = 0{\cdot}38(Gr_D Pr^2)^{1/3}$$

In the range of Gr_D from 10^8 to 3×10^8, this gives values ~ 45 per cent higher than those predicted by eq. (7). These high values may be due to the small diameter of the heated plate, leading to significantly higher local heat transfer coefficients near the plate edge.

5 RECENT MEASUREMENTS IN SODIUM

Sheriff and Davies (**1**) reported measurements made in sodium for a vertical plate 316 mm high in a pool of sodium. This work has been extended to study natural convection from downward-facing surfaces with the plate both horizontal and at an angle ~ 15 degrees to the horizontal. The arrangement of the test section is shown in Fig. 3 and a detailed description is given in reference (**1**).

For the vertical plate, experiments covered a range of modified Grashof number (used since the heat flux was uniform) of 10^6 to 3×10^{11}. Preliminary results were correlated by the following expression

$$Nu_x = 0{\cdot}674(Gr_x^* Pr^2)^{0\cdot 213}$$

For most of the Gr_x range this expression agrees within 10 per cent, which is close to the experimental accuracy, to the analytical equation (2). The positive deviations become more significant at the highest values of Gr_x, and Sheriff and Davies speculate that transition to turbulent conditions could be producing this deviation from the laminar theory. A comparison of the sodium measurements with those made in mercury is shown in Fig. 4. For low Prandtl number fluids the dimensionless group $(Gr_x^* Pr^2)$ is used as a correlating parameter, and as Fig. 4 shows there is reasonable agreement between the measurements in the two fluids. The most significant deviation occurs at the higher values of $(Gr_x^* Pr^2)$, where for sodium Gr_x^* is much larger than for mercury and could be into turbulent conditions. For comparison the theoretical difference for the two fluids is only ~ 3 per cent compared with the maximum measured difference ~ 17 per cent.

With the heater plate facing downwards at an angle of 15·2° to the horizontal, over the whole range of

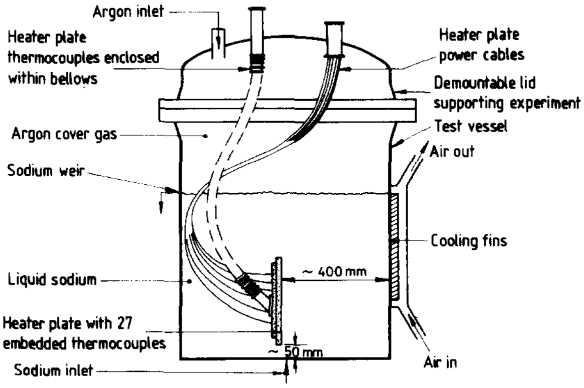

Fig. 3. Schematic arrangement of test section

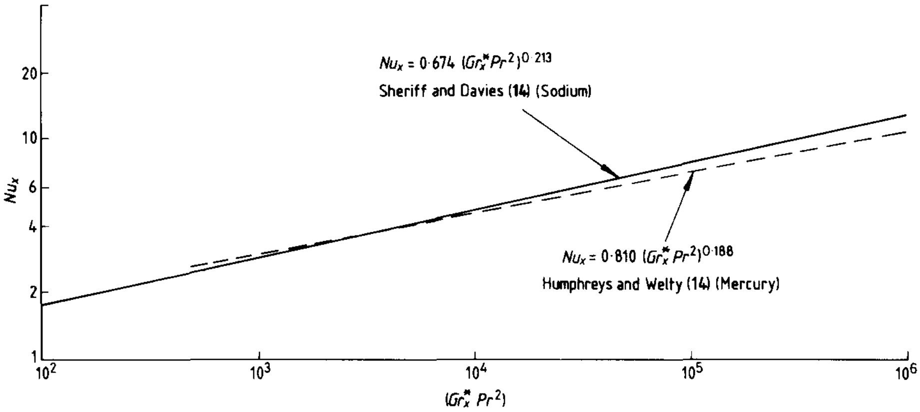

Fig. 5. Comparison of experimental sodium results with theoretical predictions

$Gr_x^* \sim 10^5$–10^{11} the experimental local Nusselt numbers were found (**23**) to be the order of 10 per cent lower than those predicted theoretically by eq. (5).

Since the vertical plate data differs by *less* than 10 per cent from eq. (2), these inclined plate results suggest that the simple ($Gr_x^* \cos\beta$) correction for inclination is not sufficiently accurate when β is large in sodium. A possible reason for this is that because of the thick boundary layers associated with sodium (~ 50 mm), for β as large as 75°, parts of the boundary layer may actually be below the leading edge of the plate.

When the heater plate was horizontal and facing downwards, for $Gr_a^* \sim 10^{10}$, experimental mean Nusselt numbers were observed (**23**) to be the order of 15 per cent higher than Fujii's theoretical prediction in eq. (4). This discrepancy may not be unreasonable considering the approximate nature of the integral method of analysis. Although there are no experimental results for horizontal downward-facing surfaces in low Prandtl number fluids with which to compare the present results, for vertical surfaces and low Prandtl numbers integral analyses predict values ~ 10 per cent lower than the more accurate similarity theory, the predictions of which have been confirmed to within 10 per cent by the vertical plate results above. Thus the trend of the present horizontal plate results compared with the integral predictions is the same as that found for vertical surfaces.

For typical experimental conditions, observed mean heat transfer coefficients for the three different inclinations are compared with the theoretical predictions in Fig. 5. The theoretical values for the vertical and horizontal are based on eqs. (2) and (4), respectively, whereas the curve for the intermediate angles is based on eq. (5).

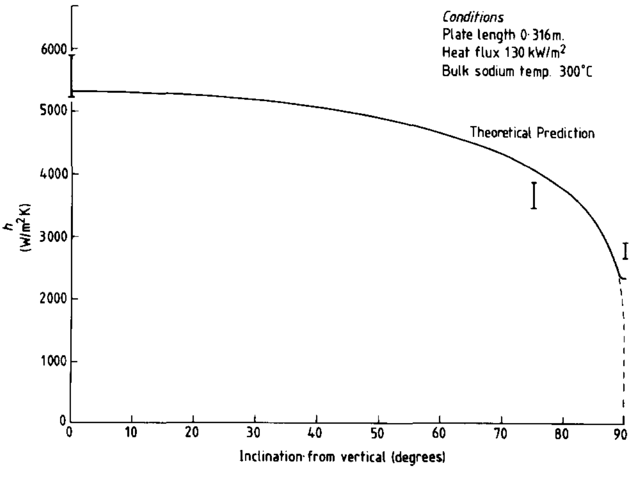

Fig. 4. Comparison of sodium and mercury results for a vertical plate

6 CONCLUDING REMARKS

Clearly from the vertical plate measurements in both mercury and sodium the laminar theoretical work quantifies liquid metal natural convection quite accurately. For other orientations tested in sodium the agreement is not as good, with discrepancies occurring above and below the measured values. The differences may not be large, however, considering the experimental uncertainties and the unknown accuracy of the theories at low Prandtl numbers. Further, the differences may not prove to be significant in practical applications.

The effect of the transition to turbulent natural convection with liquid metals was thought to have only a small effect because of the good thermal properties of liquid metals; a point first made by Bayley (**24**). Thus the extrapolation of laminar predictions beyond the transition Grashof number of other fluids was considered to be not unreasonable. Some of the sodium measurements, however, give indications that turbulence may produce significant increases just beyond the transition Grashof number. Both experimental and theoretical studies are still needed on turbulent natural convection heat transfer for liquid metals.

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